

# Using ODHeuristics To Solve Hard Mixed Integer Programming Problems

Alkis Vazacopoulos  
Robert Ashford  
Optimization Direct Inc.

February 2017

# Summary

- Challenges of Large Scale Optimization
- Exploiting parallel hardware
- The ODHeuristics approach
  - ODHeuristics Engine
  - ODH-CPLEX Optimizer
- Scheduling, supply chain and telecomms examples

# The Problem: Large Scale Optimization

- Customers and prospective customers present increasing larger and more complex models
- Standard optimization technology fails
  - Cannot find a solution at all
    - Satellite scheduling, tournament planning
  - Solutions too poor to have any value
    - Telecomms
  - Usable solutions take too long to find
    - Supply chain

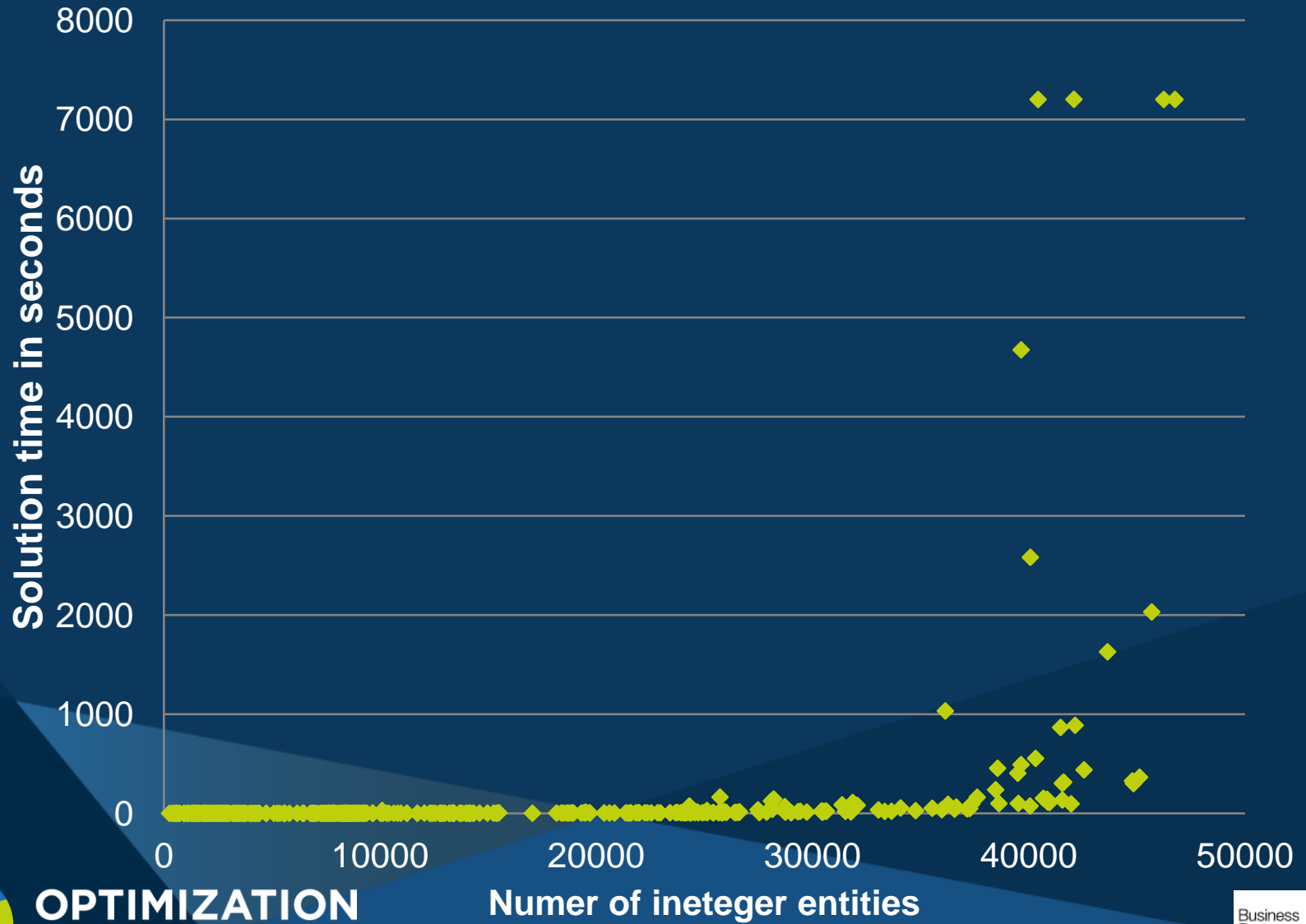
# Why Is There a Problem?

- Many models now solved routinely which would have been impossible ('unsolvable') a few years ago
- **BUT:** have super-linear growth of solving effort as model size/complexity increases
- **AND:** customer models keep getting larger
  - More and better data are available ("Big Data")
  - Globalized business has larger and more complex supply chain
  - Optimization expanding into new areas, especially scheduling
  - Detailed models easier to sell to management and end-users

# The Curse of Dimensionality: Size Matters

- Super-linear solve time growth often supposed
- **The reality is worse**
- Few data sets available to support this
- Look at randomly selected sub-models of two scheduling models
  - Simple basic model
  - More complex model with additional entity types
  - Two hour time limit on each solve
  - 8 threads on 4 core hyperthreaded Intel i7-4790K
- See how solve time varies with integers after presolve

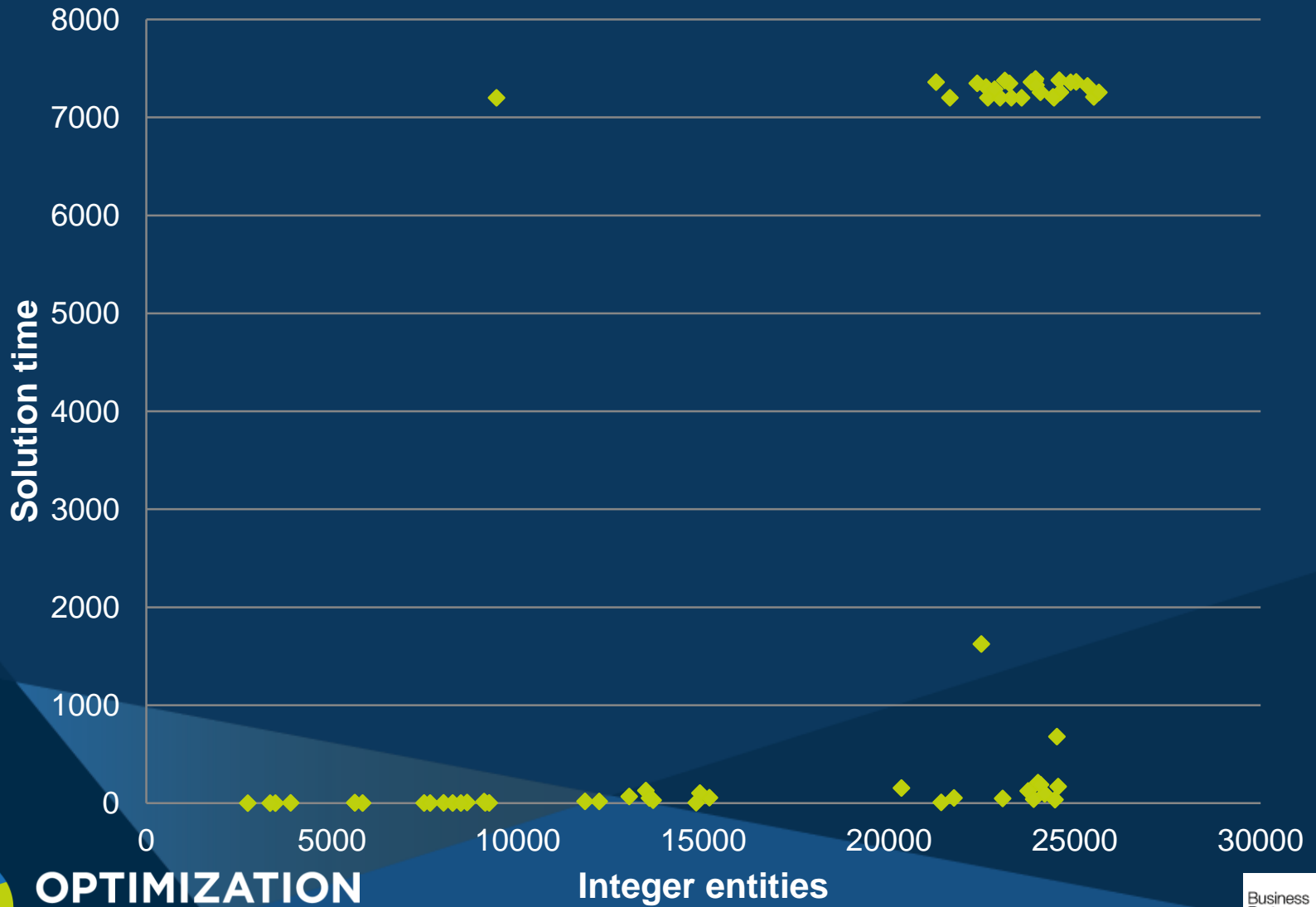
# Simple model



**OPTIMIZATION  
DIRECT**



# Complex model



# Parallelization and Hardware

- CPU clock speed is not improving
  - Power consumed (and heat generated)  $\sim$  speed<sup>2</sup>
- Memory speed is not improving (much)
- Can get wider registers (vectorization)
  - of limited use in sparse optimization
- Multi-core (processor) machines
  - Can fit more processors onto a single chip
  - 24 cores now on inexpensive servers
  - Exploited by ODHeuristics and CPLEX multi-threading
  - Cannot always use full processor capability if using many cores
  - Performance limited by bus speed ( $\sim$ 20MB/sec)



# ODHeuristics: What Is It?

- Tools for
  - handling large and/or difficult MIPs
  - exploits parallel hardware
    - typical server architecture
  - produces good solutions
    - uses CPLEX for solving sub-models
- ODHeuristics Engine
  - can be used on its own to find solutions
  - But doesn't give optimality guarantee (gap)
- ODH-CPLEX Optimizer
  - CPLEX with the ODHeuristics engine inside
  - Good at getting solutions
  - Gives optimality guarantee

# ODHeuristics Engine

- Presented as a software library
  - For embedding into customer applications
  - Call-backs and controls
  - In C, C++/Concert and Java
  - Supports Windows and Linux
- Driver programs are supplied
  - For command line use
  - As examples of calling the library
- Short User Guide (PDF)
- Skeleton scripts for compiling callers and linking

# ODHeuristics Engine: How Does it Work?

- Finds an initial solution
  - local search; and/or
  - 'bigM' and 'phase1' methods; or
  - using CPLEX
- Improves its current solution
  - Decomposes original model into sub-models
  - Finds better solution to sub-models (not necessarily optimal)
  - Each ODH thread solves its own set of sub-models
  - Combines the solutions across threads
  - Repeats with fresh decomposition
  - Progressively increases sub-model size
- Decomposition
  - Uses structure inferred from variable names and user-supplied pattern; or
  - Automatically inferred from matrix structure
- Deterministic or Opportunistic

# Examples: Large Scale Scheduling, Supply Chain and Telecomms Models

Model	entities	rows	cols	integers
Easy	314	299288	57804	57804
Medium	314	389560	94200	94200
Difficult	406	371964	149132	149132
Large	302965	2836736	4892396	1827140
Huge	27000	2577916	12944400	12944400

# Heuristic Results

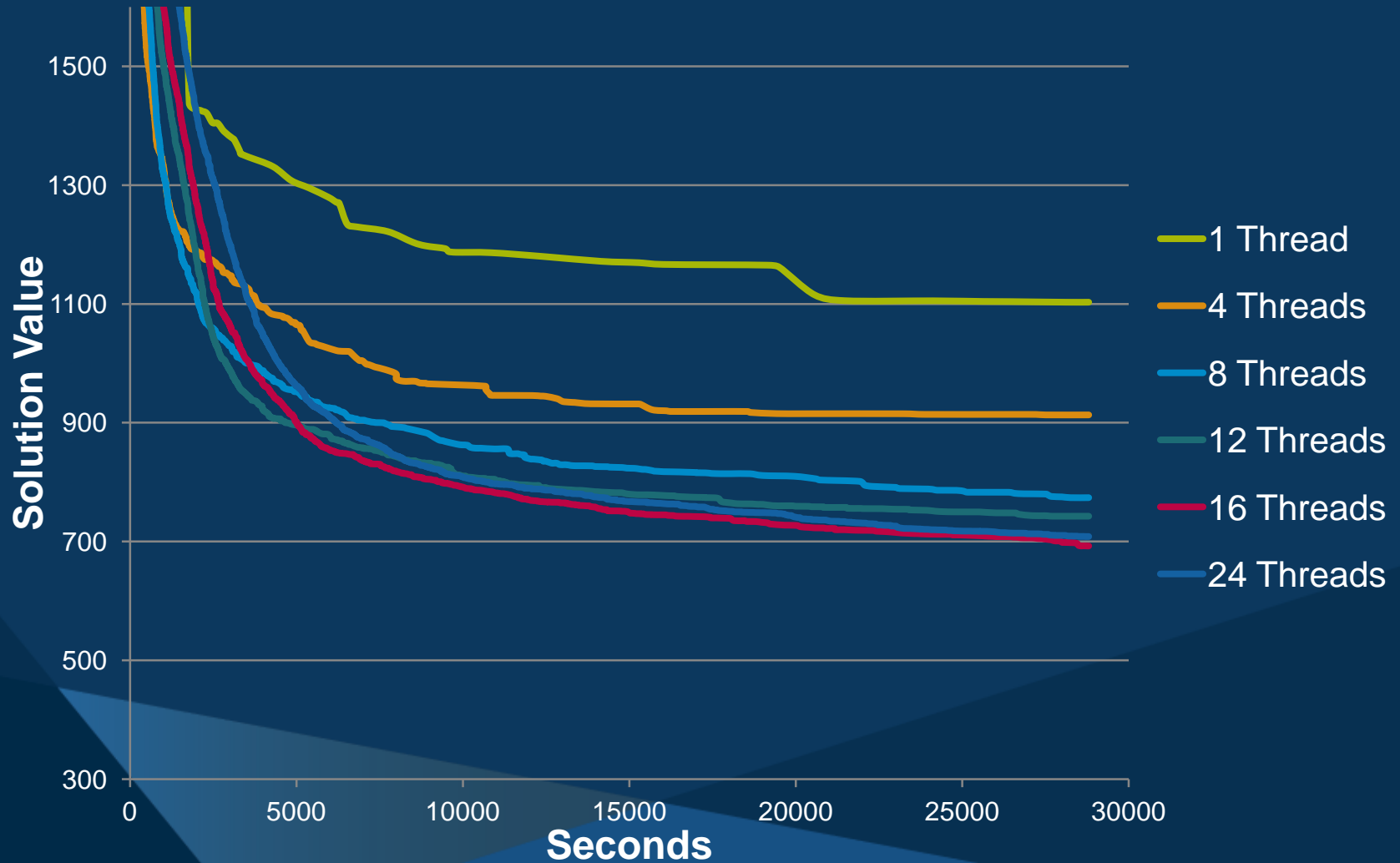
8 Threads on Intel 24 core Xeon E5-2690v3 3GHz

ODHeuristics 2.14

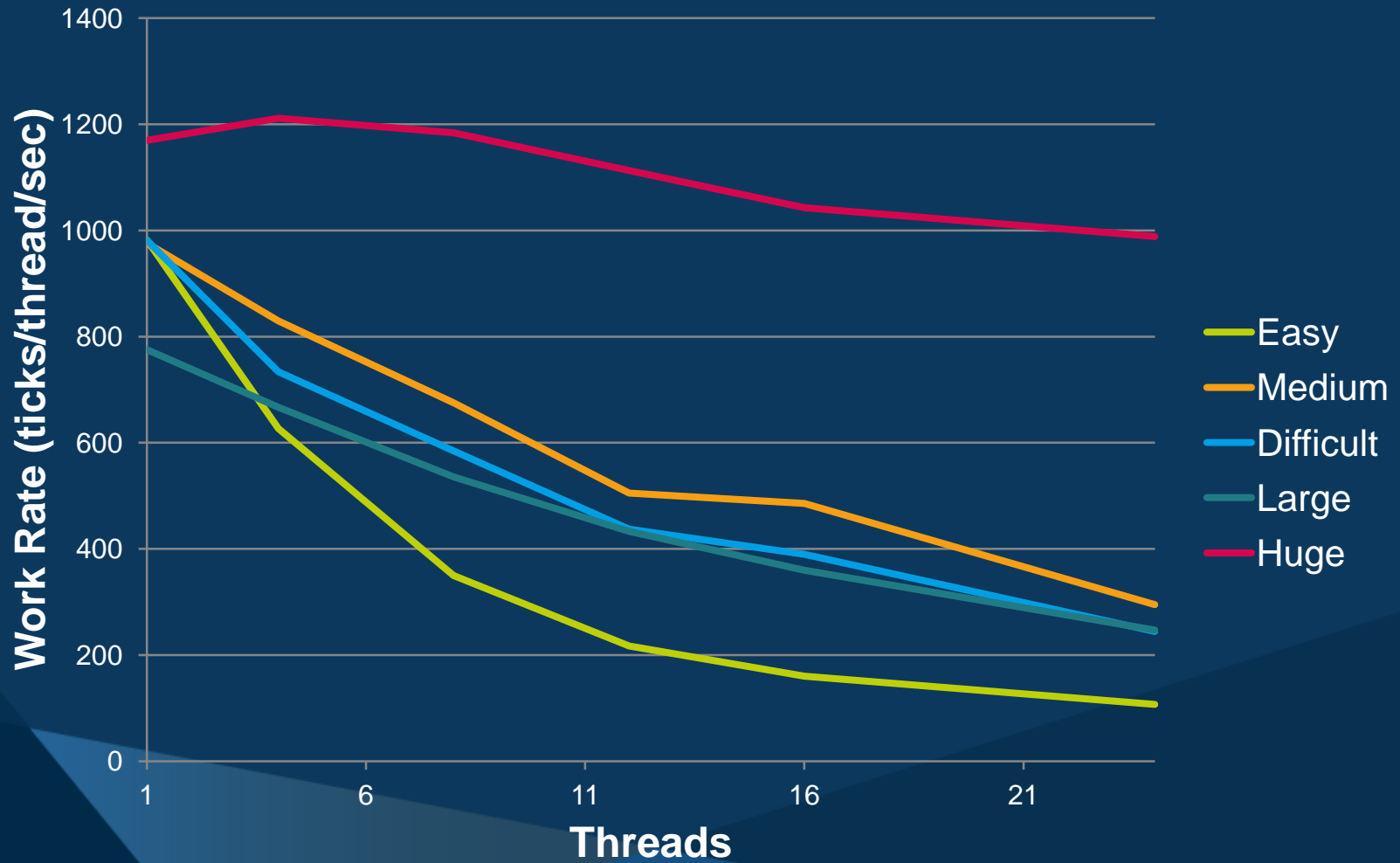
CPLEX 12.6.3

	Solution	Time	Gap	Solution	Time	Gap
Easy	96	4 mins	0%	96	4 ½ hrs	0%
Medium	113	8 hrs	≤ 13%	1161	3 ½ days	93%
Difficult	773.8	8 hrs	≤ 56%	106682	3 ½ days	99.7%
Large	1.149E+7	8 hrs	≤ 1%	1.420E+7	3 days	20%
Huge	370	8 hrs	≤ 61%	412	3 days	66%

# Difficult Model Heuristic Behavior



# Work Rate



# ODH-CPLEX Optimizer

- ODH Engine run underneath CPLEX
- Co-running continuously in separate (sets of) threads
- CPLEX initiates run of ODH
- Either can terminate and stop the other
  - time limit, gap, etc.
- Information exchanged between CPLEX and ODH Engine
- Deterministic or opportunistic modes



# ODH-CPLEX Design

- Presented as a subroutine library like CPLEX itself
- All CPLEX API calls are available including
  - Call-backs
  - Parameter interrogation and setting
- Just a different optimize call
- Behaves like CPLEX if ODH doesn't find anything useful (as much as technically possible)
- Supports ODH engine controls and call-back
- Supports C++/Concert and Java
- Supports Windows and Linux (64 bit only)

# ODH-CPLEX Optimizer Advantages

- Potentially faster than either alone
- Good solutions from ODH accelerate CPLEX, e.g:
  - reduced cost fixing
  - tree pruning
  - helps CPLEX heuristics
    - saves effort
    - provides better starting information (e.g. RINS)
- Good solutions from CPLEX accelerate ODH Engine
  - focus ODH's attention on different parts of model

# ODH-CPLEX Optimizer Challenges

- Synchronization overheads in deterministic mode
  - For CPLEX up to 20% but usually less than 5%
- CPLEX solution paths are changed by just the presence of communication channel with ODH (even if no information is exchanged)
- Needs more memory than CPLEX or ODH alone
  - Twice as much
- Needs more processors/cores
  - 8+ cores to run well, 24+ cores ideal
  - Hyperthreaded (SMT) core splitting not a good idea
- Increases demand on memory bus
  - But less than if all threads were committed to ODH alone

# ODH-CPLEX Results

- Similar to CPLEX alone on MIPLIB2010
  - Tested on subsets used by Mittelmann
  - 4 threads and 24 threads
  - Resources spent on ODH engine compensate for loss to main CPLEX solver
    - threads and memory bus contentions
  - Comparisons done with same number of threads in total
- Usually very beneficial on hard user models

# ODH-CPLEX Results

24 Threads on Intel 24 core Xeon E5-2690v3 3GHz

	CPLEX 12.7.0		ODH-CPLEX Optimizer 3.00	
	24 threads		18 (CPLEX) + 6 (ODH) threads	
	Time	Gap	Time	Gap
Easy	2 hrs	3.73%	2 hrs	16.67%
Medium	8 hrs	$\infty$	2 hrs	36.36%
Difficult	8 hrs	$\infty$	2 hrs	74.70%
Large	8 hrs	8.05%	2 hrs	0.81%

# ODH-CPLEX Results: Recent User Models

Model	rows	cols	nonzeros	binaries	integers
Accrue	126367	107788	431668	23652	0
Balance	732311	719937	3384573	664678	0
Fact	343629	336960	1538418	299656	0
General	813225	675862	2953834	25518	14466
Hands	315763	153037	2537381	82723	0
Joint	138562	109964	552333	69000	0
Keep	108675	81488	415981	54000	0
Latter	395435	543700	2235636	8695	7050
Made	395439	543700	2235640	8695	7050
Obligation	45840	70697	263418	10944	4119
Paid	579250	509399	2457594	72202	0
Rank	866477	1176512	4788339	87721	1
Said	40474	44304	177842	9552	4807
Valid	37224	65845	211514	6979	5909
Wales	105704	79070	421342	52500	0

# ODH-CPLEX Results: Recent User Models

Model	CPLEX		ODH-CPLEX	
	Time	Gap	Time	Gap
Accrue	678.07	0%	229.78	0%
Balance	7202.58	8.52	7203.63	6.85%
Fact	7201.28	6.52%	7201.21	5.25%
General	7243.64	89.52%	7218.71	7.86%
Hands	7201.32	inf	7201.81	inf
Joint	7200.20	93.65%	7200.23	26.79%
Keep	7200.68	26.76%	7202.22	14.70%
Latter	571.82	0%	521.79	0%
Made	238.25	0%	73.52	0%
Obligation	7.19	0%	4.35	0%
Paid	7200.55	21.87%	7216.42	0.49%
Rank	267.10	0%	397.29	0%
Said	7222.21	4.57%	7214.97	6.23%
Valid	0.20	0%	0.35	0%
Wales	7200.68	9.86%	7201.20	5.82%

# Conclusions

- Customers now want to solve larger and large models
- Hard size barriers to solve (to optimality) or even to getting a solution at all
- ODHeuristics can find good solutions
  - Useful on small(er) models too
- ODH-CPLEX can provide solutions of proven optimality quality
- Parallel solution methods best way of exploiting modern hardware (although limited by memory bus speeds)



# Benchmarking and Evaluation

- If you think that ODHeuristics and/or ODH-CPLEX might work for you:
- send us your difficult matrices and we will send you the results
- request an evaluation copy

# Thanks for listening

Robert Ashford

[rwa@optimizationdirect.com](mailto:rwa@optimizationdirect.com)

[www.optimizationdirect.com](http://www.optimizationdirect.com)

